A LOCAL DISCONTINUOUS GALERKIN METHOD FOR THE NAVIER-STOKES-KORTEWEG EQUATIONS

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Key words: *Navier-Stokes-Korteweg equations, local discontinuous Galerkin method, phase transition*

Diffuse interface methods provide an interesting way to compute multiphase flows since only a single set of equations is used. There is no need to explicitly compute the interface between the phases as in Volume of Fluid or Level Set methods. In this presentation we will consider the Navier-Stokes-Korteweg (NSK) equations to compute phase transition between liquid and vapour in compressible fluids. These equations contain, next to the viscous stress tensor, also the Korteweg tensor, and are suitable for a diffuse interface method. Both the isothermal and non-isothermal NSK equations will be discussed. The NSK equations are closed using the Van der Waals equation of state and phase transitions between liquid and vapour are distinguished using different values of the density. The addition of the Korteweg tensor to the Navier-Stokes equations results in a system of third order partial differential equations. This induces a dispersive behaviour in the numerical solution, which might cause numerical instabilities in the phase transition region. Also, during phase transition the equations can locally change type. In this presentation we will present a new local discontinuous Galerkin discretisation for the NSK equations. This LDG discretisation was motivated by our recent research on a hyperbolic-elliptic system of partial differential equations modelling phase transition in solids and fluids, [1]. An important feature of the LDG discretisation that we will present for the NSK equations is that it uses the conservative form of the NSK equations. The LDG discretisation is therefore conservative and also satisfies the energy decay relation, which is violated in some existing numerical schemes. Our approach is also suitable for the non-isothermal NSK equations. In contrast, many existing schemes for the isothermal NSK equations can not be easily extended to the non-isothermal case, since they require that the pressure only depends on the density, which is not the case for the non-isothermal NSK equations. Also, these methods frequently require extra stabilisation terms to ensure numerical stability.

This is not necessary for the LDG discretisation. Since the LDG discretisation of the NSK equations results in a stiff system of ordinary differential equations, we use a Singly Diagonally Implicit Runge-Kutta method for the time integration. This removes the rather stringent time-step restriction at a reasonable computational cost. The resulting algebraic equations from the SDIRK method are solved with a Newton method in combination with a GMRES Krylov solver with an incomplete LU decomposition as preconditioner. The LDG discretisation for the NSK equations will be demonstrated on a number of test cases. First, results of 1D and 2D model problems will be used to demonstrate that the LDG discretisation achieves optimal order of accuracy and we investigate its behaviour in the phase transition region. Next, the potential of the LDG discretisation for practical phase transition problems will be demonstrated with simulations of the coalescence of two bubbles. Both the isothermal and non-isothermal case will be considered. For more details, see [2].

REFERENCES

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